Y12 Lesson 2 Activities

In this worksheet we will consider the energy changes occurring as the air above the ground surface cools overnight. This explains why the temperature difference between evenings and mornings in winter is often greater than during the summer, as there are fewer hours of daylight in winter.

The model used to investigate these energy changes is *Newton's Cooling Law*, which states that the rate of change of the temperature with respect to time of a non-heated body is proportional to the difference between the body's temperature and the ambient temperature of the surroundings.

1) Suppose that **T** represents temperature in °C, **t** represents time in seconds and $\mathbf{T_A}$ represents the ambient temperature of the surroundings. How could this be written as a first order differential equation?

2) Find the general solution to the first order differential equation in question 1) by using the method of integrating factors. *[Hint:* $\frac{d}{dt}\left(Te^{kt}\right) = e^{kt}(\frac{dT}{dt} + kT)$ *]*

3) Now, consider a weather model where the ground level air is heated by the sun during daylight hours and cools according to Newton's Cooling Law overnight. In this model we assume the temperature of the ground remains at 10°C overnight (hence $T_A = 10$ throughout).
   a) Given that the ground level air temperature decreases from 20°C to 12°C between 9pm and 5am on one night, find the value of **k** in this particular solution.
   b) Find the time taken for the temperature to reach 15°C for the model in part a).
   c) On the following night at 9pm the temperature is 24°C, find the expected temperature 8 hours later (5am in the morning) using the value of **k** from part a). *[Hint: You will need to re-calculate the integration constant as the initial condition has changed!]*
   d) The next morning at 5am the temperature is observed to be 10.2°C, by setting this as an initial condition find the expected temperature 5 hours earlier at midnight. *[Hint: Use a negative time to represent an event occurring in the past.]*

4) Are the assumptions made in question 3 justifiable and how could this model be improved?

Alexander Todd, University of Exeter
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