Y12 Lesson 2

Overview:
This lesson builds on the concepts introduced in the first lesson to construct a simplified case of an atmospheric model. The key themes covered in this lesson are:

- Conservation of Energy,
- First Order Differential Equations,
- Method of integrating factors and Integration by parts (Extension material)

Introduction (10 – 15 minutes presentation):
Start the lesson by displaying a video such as https://www.youtube.com/watch?v=DFcuZka0LLA which shows global cloud circulation in 2011. Ask students if they can identify any patterns, explain how weather forecasters can use equations to predict how these patterns will change over time.

Lesson Plan:
* = activity on individual whiteboards/verbal activity
**=more advanced activities for higher sets

1. **Conservation of Energy:** Introduce the concept of how the atmosphere can be thought of as a system with energy inputs and outputs. The total energy input must equal the total energy output.
   1.1. Use the diagram at http://www.ipcc.ch/publications_and_data/ar4/wg1/en/faq-1-1-figure-1.html to demonstrate the complexity of these interactions.

2. **Conservation of Energy Example:** Work through example of conversion between gravitational potential energy and kinetic energy.
   2.1. Use a diagram to show how we can calculate the Gravitational Potential Energy $\mathbf{E_G}$ of a particle of mass **m** in kg of height **h** in m above the Earth's surface, where **g** = $9.8ms^{-2}$ is the acceleration due to gravity. ($\mathbf{E_G=mgh}$)
   2.2. State how we can calculate the Kinetic Energy $\mathbf{E_K}$ of a particle of mass **m** in kg moving with speed **v** in ms^{-1} by using $\mathbf{E_K=mv^2/2}$
   2.3. ***Using whiteboards ask students to work through a few simple examples (invent reasonable values) such as:
      2.3.1. Calculating the speed of a particle falling from a fixed height,
      2.3.2. Calculating the height a particle will reach starting with an initial speed. (*Initial Value Problem*)

3. **Newton's Cooling Law:** Introduce the simplified cooling problem:
   3.1. Suppose the air above the ground surface is warmed during daylight hours by incoming radiation from the sun. During the night the air cools significantly more quickly than the ground surface (differences in specific heat capacity).
   3.2. We assume that the air is still and the ground remains the same temperature overnight.
   3.3. Introduce Newton's Cooling Law (top of worksheet) and ask students how it could be written as a first order differential equation (1st question on worksheet)
   3.4. Solution is: $\frac{d\boldsymbol{T}}{d\boldsymbol{t}} = -\boldsymbol{k}(\boldsymbol{T}-\boldsymbol{T_A})$

4. **Worksheet:** The first question involves the method of integrating factors; this is suitable for Further Maths sets but should be worked through on the whiteboards as a class for Maths sets.
   4.1. Ask students to complete the worksheet and mark together as a class.

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5. **More complex Cooling Law:** Introduce a more complex cooling model where the rate of change of temperature is proportional to the difference between the ambient and current temperature and also the time. This can be represented by the 1st order differential equation: $\frac{dT}{dt} = k(T_A - T) - ct$

   5.1. **Demonstrate how we can solve this to find the general solution:

      5.1.1. First use method of integrating factors to obtain: $\frac{d}{dt}\left(Te^{kt}\right) = e^{kt}(kT_A - ct)$

      5.1.2. Then apply integration by parts where: $u = kT_A - ct$ and $v' = e^{kt}$

      5.1.3. This gives: $T(t) = T_A - \frac{ct}{k} + \frac{c}{k^2} + De^{-kt}$

   5.2. Using the interactive graph tool available at https://www.desmos.com/calculator/eyjljfkvtu to display how we can vary the constants in the general solution to produce a family of solution curves.

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