

Y12 Lesson 1 Activities

In this worksheet we will consider the movement of a *super-cell cumulonimbus* cloud moving across the UK which brings heavy rain, hail, thunder and lightning. These clouds form when warm, humid air rises quickly, condensing to form turbulent columns of water droplets up to 16km high.

Motion in 1-D

- 1) Consider a ball moving on a smooth level surface with displacement $x(t) = 2t^2 - 4t$ m at time t in seconds,
 - a) Find expressions for the velocity and acceleration of the particle at time t .
 - b) Find the displacement, velocity and acceleration at time $t = 30$.
 - c) Find the times at which the particle has zero displacement and also the time at which the particle is stationary.

- 2) A stationary *super-cell cumulonimbus* has a base 3km above a level ground surface. Given that acceleration due to gravity is $\mathbf{g} = 9.8\text{ms}^{-2}$ vertically downwards,
 - a) Sketch a diagram representing the cloud base, the ground surface with height x m increasing vertically upwards from the ground surface.
 - i) Why is it suitable to model a hail stone falling from the cloud as a point particle?
 - b) Find an expression in \mathbf{g} for the velocity of a hail stone, $\frac{dx}{dt}$ in terms of time t given that the hail stone starts from rest at the base of the cloud and falls vertically down towards the ground surface. [Hint: $\frac{d^2x}{dt^2} = -g$]
 - c) Using your solution to part b), find an expression in \mathbf{g} for the height x m of the hail stone above the ground surface in terms of time t which falls from rest from the base of the cloud.
 - d) Using your solutions to parts b) and c), find the time to 3 s.f. at which the hail stone hits the ground, and the velocity of impact.
 - e) Now, suppose the moves over a mountain which is 1.5km high, so this distance between the cloud base and ground surface decreases to 1.5km. Find the time to 3 s.f. at which the hail stone hits the ground, and the velocity of impact assuming the hail stone falls from rest from the cloud base.

Motion in 2-D

- 1) A *super-cell cumulonimbus* is moving with constant velocity from Birmingham to London. Birmingham is situated 100km north and 170km west of London
 - a) Sketch the positions of Birmingham and London on a graph where the origin is 100km south of Birmingham and 170km east of London.
 - b) Find the speed and velocity of the cloud in terms of time t in kmh^{-1} given that it takes 4 hours for the cloud to reach London.

- 2) Now assume that the cloud starts from rest at the origin and is moving with a changing velocity where the position at time t is given by: $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$
 - a) Suppose the velocity of the cloud at time t is given by: $\frac{d\mathbf{r}}{dt} = 3\mathbf{i} + 4t\mathbf{j} \text{ kmh}^{-1}$, find the displacement and acceleration of the cloud by considering each component in turn. Hence write down the functions $x(t)$ and $y(t)$.
 - b) Find the time taken for the cloud to move to the location with the y co-ordinate 400km and find the x co-ordinate at this time.
 - c) Show that the path of the cloud will be defined by the curve $y = \frac{2x^2}{9}$

Further Questions

- 1) Suppose the velocity of a cloud at time t is given by: $\frac{dr}{dt} = 3t^2\mathbf{i} + 4t\mathbf{j} \text{ kmh}^{-1}$,
 - a) Find the acceleration and displacement of this cloud given that it is at rest at the origin when $t = 0$.
 - b) Find the path of the cloud given that the position at time t is given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$
 - c) Will the cloud pass through the point $3\mathbf{i} + 4\mathbf{j}$?

- 2) Now consider a cloud moving in 3 dimensions: x represents east-west, y represents north-south and z represents vertical displacement above the Earth's surface; the displacement at time t given by: $\mathbf{r}(t) = 2t^3\mathbf{i} + 3\mathbf{j} + t\mathbf{k}$.
 - a) Given that $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ find the equations for $x(t)$, $y(t)$ and $z(t)$ and hence write x in terms of z .
 - b) Describe what the path of this cloud would look like.